Odds, Evens and Proofs

Fill in the table:

+	odd	even
odd	e	o
even	o	e

To Prove:

o + o = e

An even number can be expressed as $2n$

So an odd no can be expressed as $2n+1$

Prove it!

$$\text{odd} + \text{odd} = (2n+1) + (2m+1)$$
$$= 2n + 2m + 2$$
$$= 2(n+m+1)$$

which is even

QED

Look at the pattern:

$20^2 = 400$	$19 \times 21 = 399$
$30^2 = 900$	$29 \times 31 = 899$
$40^2 = 1600$	$39 \times 41 = 1599$
$50^2 = 2500$	$49 \times 51 = 2499$
$60^2 = 3600$	$59 \times 61 = 3599$
n^2	$(n-1)(n+1) = n^2 - 1$

Continue the pattern for 3 more rows.
Write an expression for the nth row.
Prove why this works.

$$\text{LHS } (n-1)(n+1) = n^2 + n - n - 1$$
$$= n^2 - 1$$
$$= \text{RHS}$$

QED

State whether this is true of false, and give a convincing argument....

The product of two even numbers is even.

T

To prove the product of two even nos is even

Let one even no be $2n$ and the other $2m$

$$2n \times 2m = 4nm$$
$$= 2(2nm) \text{ which is even}$$

QED

State whether this is true of false, and give a convincing argument....

The sum of any three consecutive numbers is divisible by 3.

$n-1,\ n,\ n+1$

$$3n+1-1=3n$$

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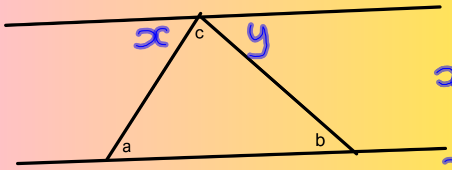
Geometric proof

Given that the angles on a straight line total 180^0, prove that the angle sum of a triangle is 180^0.



To prove $a+b+c=180^\circ$

$x+c+y=180^\circ$ (given)

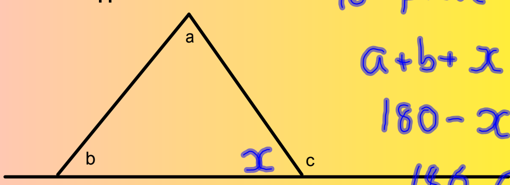
$x=a$ (alternate angles)

$y=b$ (alternate angles)

$\therefore\ a+c+b=180^\circ$ QED.

Prove that the external angle of a triangle is equal to the sum of the interior opposite angles.

Prove that the interior angle of an n sided regular polygon is equal to $\frac{180(n-2)}{n}$



To prove $a+b=c$

$a+b+x=180^\circ$ (Δ) ②

$180-x=c$

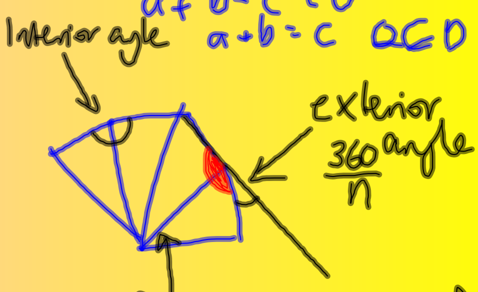
$180-c=x$ ①

$a+b+(180-c)=180$

$a+b+\cancel{180}-c=\cancel{180}$

$a+b-c=0$

$a+b=c$ QED



$\frac{180(n-2)}{n}$

$\frac{180\cancel{n}}{\cancel{n}}-\frac{360}{n}$

$180-\frac{360}{n}$

Ext angle $=\frac{360}{n}$ (angle at centre)

Int angle $=180-\frac{360}{n}$

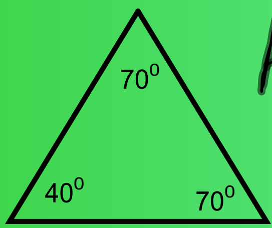
$=\frac{180n}{n}-\frac{360}{n}$

$=\frac{180}{n}(n-2)$

Triangles

Accurately draw these triangles

Using a different colour measure all missing elements and mark in their length/size.



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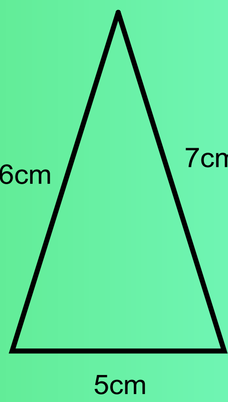
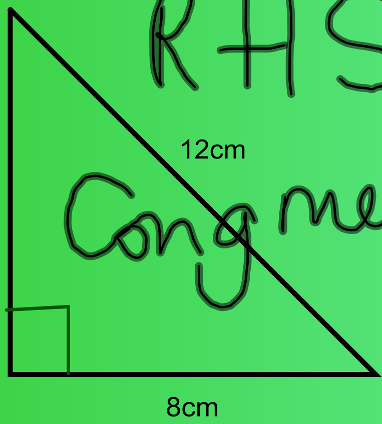


SSS

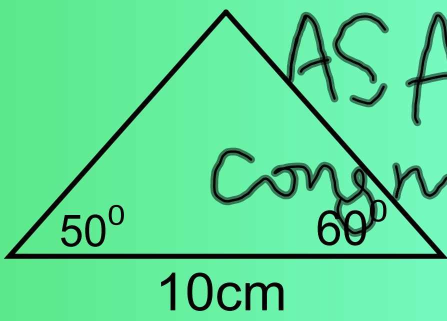
Congruent

RHS



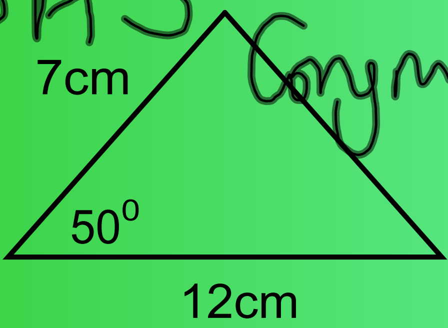
Congruent

ASA



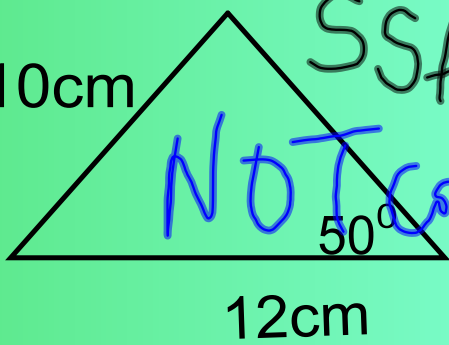
Congruent

SAS



Congruent

SSA



NOT Cong

Congruency

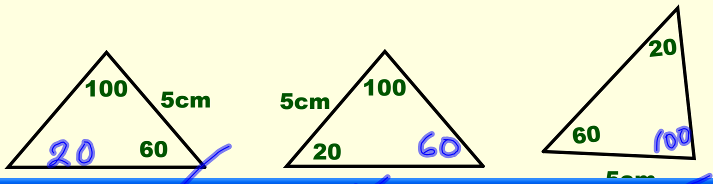
Triangles are congruent if......

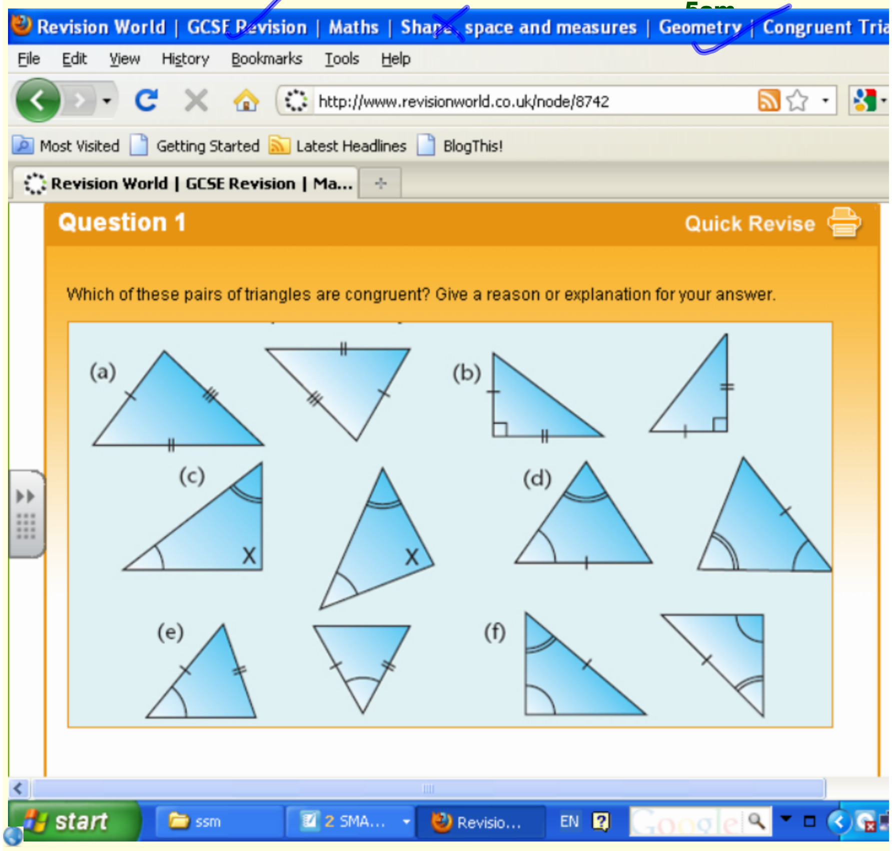
RHS

SSS

SAS (order is important)

ASA **(order not so important but the sides must be corresponding)**





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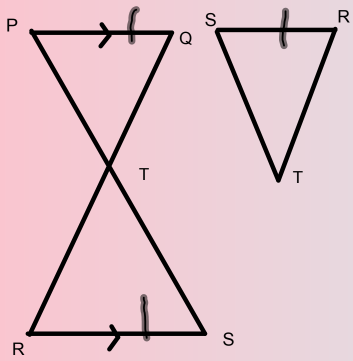
Proofs and congruency



$\angle QPT$

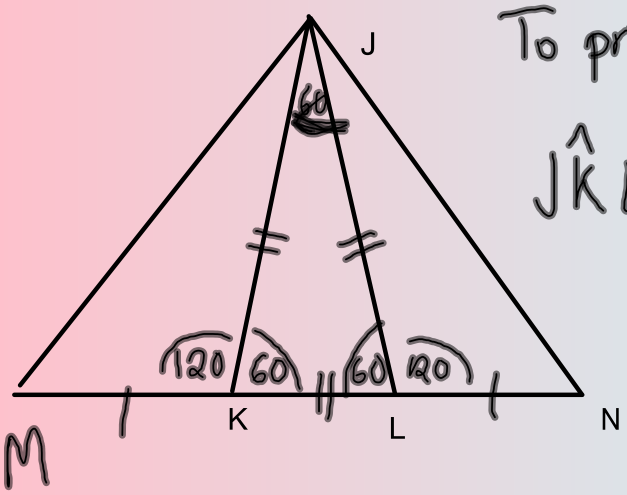
To prove $\triangle PQT \equiv \triangle SRT$

$PQ = SR$ (given)

$Q\hat{P}T = T\hat{S}R$ (alternate angles are equal)

$P\hat{Q}T = T\hat{R}S$ (")

$\therefore \triangle PTQ \equiv \triangle STR$ (ASA)



To prove: $\triangle JKM \equiv \triangle JLN$

$J\hat{K}L = 60^\circ = J\hat{L}K$ (given)

$\therefore J\hat{K}M = J\hat{L}N = 120^\circ$ (angles on a straight line)

SAS RHS AAS SSS

$KM = LN$ (given)

$JK = LJ$ (given)

$\therefore \triangle JKM \equiv \triangle JLN$ (SAS)

QED

Red book page 49
Q1 onwards or
Q 2,and then Q5 onwards

Proofs using congruency

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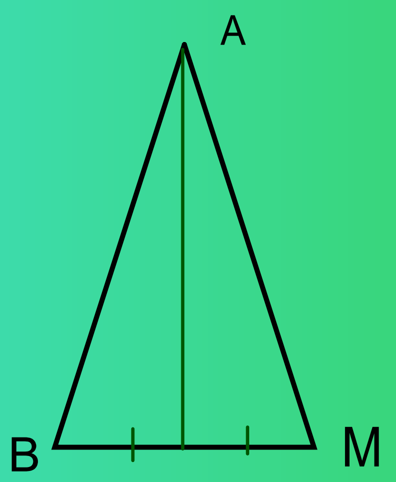

Triangle XYZ is isosceles with XY = XZ. The bisector of angle Y meets XZ at M; the bisector of angle Z meets XY at N. Prove that YM = ZN.

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